



Dice analysis report

for PartyGammon system

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Executive Summary

iTech Labs have calculated theoretical probabilities for a number of dice outcomes in Backgammon Games and compared these against corresponding probabilities for dice throws in real money games in the PartyGammon system. We have done calculations for the following dice outcomes: [Any Specific Number](#), [Sum of Pairs](#), [Matching Pairs](#), [Un-matching Pairs](#) and [Two Identical Rolls](#).

The calculations were done for a total of 55,975,440 dice numbers (27,987,720 dice throws). The chi-squared statistic and p-values for the dice outcomes showed statistical randomness. The observed values were within 95% confidence limits of the theoretical values, except for minor exceptions that were very well within the acceptable limits. There is thus no evidence of non-randomness in the sequence of dice throws. iTech Labs concludes that the Random Number Generator (RNG) used for the games in the PartyGammon system is operating correctly.

iTech Labs had previously tested and certified the RNG used in the PartyGammon system. The findings from the analysis of dice numbers from the live system confirms that the random numbers generated by the RNG are being used in a fair manner resulting in statistically random rolls of dice.



1. Introduction

This report is aimed at providing theoretical probabilities and corresponding actual probabilities for various outcomes of PartyGammon Games. iTech Labs have calculated theoretical probabilities for a number of dice outcomes in Backgammon Games and compared these against corresponding probabilities for dice throws in real money Games in the PartyGammon system.

The calculations were done for **Any Specific Number, Sum of Pairs, Matching Pairs, Un-matching Pairs and Two Identical Rolls**. Section 2 shows the calculations for all dice numbers captured from the system, irrespective of the Game type or the Board. We have also done similar calculations for Boards. All games played on a Board were included in the Board specific calculations. Section 3 shows calculations conducted for a typical Board.

The actual probability calculations for real money games were done for a total of 55,975,440 dice numbers (27,987,720 dice throws). We have captured this data from gamelogs obtained from the game servers. iTech labs have also captured a limited number of dice data manually from games played by us for validating the results obtained from the sever logs. In both cases, the results were within 95% confidence limits.

It should be noted that the analysis was done for groups of twenty throws which were not carried over to the next twenty. This means that there were 19 pairs and 17 groups of 4 for each 20 throws (e.g., The pairs in 20 throws are 1,2; 2,3; 3,4;...; 19,20. The groups of 4 are 1,2,3,4; 2,3,4,5; ...; 17,18,19,20. Thus there are 19 sets of pairs and 17 groups of 4 in each set of twenty throws). This method of selection of pairs and groups in no way biases the results.

2. Probabilities for 55,975,440 dice numbers (27,987,720 dice throws)

The dice throws included for calculations in this section were captured from server gamelogs for Backgammon, Hypergammon and Nackgammon in the following formats:

- Single game
- Match
- Sit and Go
- Tournaments (Regular)

All dice numbers captured from the system, irrespective of the Game type or the Board were included in these calculations.

2.1 Any Specific Number

The following table compares the observed proportion of times various events occurred (**Sample** column) against theoretical probabilities (**Probability** column). **Lower** and **Upper** are the 95% confidence limits.

Face	Probability	Expected	Lower	Sample	Upper	Ratio
1	1/6	9329240	9323663	9328592	9334817	0.9999
2	1/6	9329240	9323663	9333444	9334817	1.0005
3	1/6	9329240	9323663	9331415	9334817	1.0002
4	1/6	9329240	9323663	9328798	9334817	1.0000
5	1/6	9329240	9323663	9328341	9334817	0.9999
6	1/6	9329240	9323663	9324850	9334817	0.9995

Chi-squared statistic = 4.6 Not significant p=0.46

2.2 Sum of Pairs

Sum	Probability	Expected	Lower	Sample	Upper
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2	1/36	1477130	1474733	1477915	1479526
3	2/36	2954259	2950919	2953285	2957600
4	3/36	4431389	4427358	4434068	4435420
5	4/36	5908519	5903935	5909130	5913102
6	5/36	7385648	7380605	7386039	7390692
7	6/36	8862778	8857343	8863056	8868213
8	5/36	7385648	7380605	7389375	7390692
9	4/36	5908519	5903935	5907302	5913102
10	3/36	4431389	4427358	4427719	4435420
11	2/36	2954259	2950919	2951418	2957600
12	1/36	1477130	1474733	1477361	1479526

Chi-squared statistic = 10.4 Not significant p=0.4

2.3 Matching Pairs

Pair	Probability	Expected	Lower	Sample	Upper
1+1	1/36	1477130	1474733	1477915	1479526
2+2	1/36	1477130	1474733	1480180	1479526
3+3	1/36	1477130	1474733	1478405	1479526
4+4	1/36	1477130	1474733	1477090	1479526
5+5	1/36	1477130	1474733	1476443	1479526
6+6	1/36	1477130	1474733	1477361	1479526

Chi-squared statistic = 8.17 Not significant p=0.15

2.4 Un-matching Pairs

First	Second	Probability	Expected	Lower	Sample	Upper
1	2	1/36	1477130	1474733	1477692	1479526
1	3	1/36	1477130	1474733	1475633	1479526
1	4	1/36	1477130	1474733	1477681	1479526
1	5	1/36	1477130	1474733	1475965	1479526
1	6	1/36	1477130	1474733	1477315	1479526
2	1	1/36	1477130	1474733	1475593	1479526
2	3	1/36	1477130	1474733	1478302	1479526
2	4	1/36	1477130	1474733	1476671	1479526
2	5	1/36	1477130	1474733	1476923	1479526
2	6	1/36	1477130	1474733	1477961	1479526
3	1	1/36	1477130	1474733	1478255	1479526
3	2	1/36	1477130	1474733	1475720	1479526
3	4	1/36	1477130	1474733	1477015	1479526
3	5	1/36	1477130	1474733	1478354	1479526
3	6	1/36	1477130	1474733	1477062	1479526
4	1	1/36	1477130	1474733	1477427	1479526
4	2	1/36	1477130	1474733	1478097	1479526
4	3	1/36	1477130	1474733	1478448	1479526



4	5	1/36	1477130	1474733	1478290	1479526
4	6	1/36	1477130	1474733	1473405	1479526
5	1	1/36	1477130	1474733	1476901	1479526
5	2	1/36	1477130	1474733	1478179	1479526
5	3	1/36	1477130	1474733	1478170	1479526
5	4	1/36	1477130	1474733	1476458	1479526
5	6	1/36	1477130	1474733	1475909	1479526
6	1	1/36	1477130	1474733	1475176	1479526
6	2	1/36	1477130	1474733	1477800	1479526
6	3	1/36	1477130	1474733	1475492	1479526
6	4	1/36	1477130	1474733	1477871	1479526
6	5	1/36	1477130	1474733	1475509	1479526

Chi-squared statistic = 31.3 Not significant p=0.36

2.5 Two Identical Rolls

The identical rolls have been lumped because the sample size is too small for individual sequences.

Pairs	Probability	Expected	Lower	Sample	Upper
1+1 ; 1+1	1/1296	36712	36329	36798	37095
2+2 ; 2+2	1/1296	36712	36329	36709	37095
3+3 ; 3+3	1/1296	36712	36329	36539	37095
4+4 ; 4+4	1/1296	36712	36329	36407	37095
5+5 ; 5+5	1/1296	36712	36329	36828	37095
6+6 ; 6+6	1/1296	36712	36329	36874	37095

Chi-squared statistic = 4.63 Not significant p=0.46

Notes:

- Actual dice numbers from PartyGammon system is shown in this shade
- Theoretical probability of 1/6 for a face (dice number) means, players are likely to get any single face 1/6 of the total number of dice numbers.
- When the sample size is not small, the formula

$$p - 1.96\sqrt{\frac{p(1-p)}{n}}, p + 1.96\sqrt{\frac{p(1-p)}{n}}$$

gives approximate 95% confidence limits for the probability, where p =theoretical probability, n =total dice numbers. The lower and upper limits in the tables above are obtained by multiplying the total number of dice numbers with the lower and upper probabilities respectively.

- The chi-squared statistic is a measure of how close the observed numbers are to the expected numbers. If there are significant differences between the theoretical probabilities and the actual probabilities, one would expect the chi-squared statistic to be very large when dealing with 27.987720 million dice throws.
- The p -value is the probability one would have got if the random number generator was working properly. In general no notice is taken of the p -value unless it is below 5% (0.05). 5% means that there is still a 1/20 chance that we would have got this data with the random number generator working properly. For over 27 million dice throws, even small deviations from randomness will show up as small p -values (say <0.001).



f) In the above tables, 2 numbers are shown as bold. These two numbers are outside the lower/upper limits. Up to 5% of the numbers can be outside the limits.

Comments:

In the above tables for calculations of all dice throws irrespective of game type or board, 96.29% of actual values lie within the lower and upper limits. None of the table has a significant p-value. This shows that the Random Number Generator (RNG) is working correctly.

3 Probabilities for 25302 dice numbers (12651 dice throws) - Board specific calculations

This section shows calculations for a typical Board. All dice throws included for calculations in this section were captured from a single Board.

3.1 Any Specific Number

The following table compares the observed proportion of times various events occurred (**Sample** column) against theoretical probabilities (**Probability** column). **Lower** and **Upper** are the 95% confidence limits.

Face	Probability	Expected	Lower	Sample	Upper	Ratio
1	1/6	4217	4098	4151	4335	0.984
2	1/6	4217	4098	4300	4335	1.020
3	1/6	4217	4098	4242	4335	1.006
4	1/6	4217	4098	4263	4335	1.011
5	1/6	4217	4098	4242	4335	1.006
6	1/6	4217	4098	4102	4335	0.973

Chi-squared statistic = 6.6 Not significant p=0.25

3.2 Sum of Pairs

Sum	Probability	Expected	Lower	Sample	Upper
2	1/36	668	617	636	719
3	2/36	1335	1264	1311	1406
4	3/36	2003	1917	2027	2089
5	4/36	2671	2573	2724	2768
6	5/36	3338	3231	3393	3445
7	6/36	4006	3890	3986	4121
8	5/36	3338	3231	3438	3445
9	4/36	2671	2573	2604	2768
10	3/36	2003	1917	1964	2089
11	2/36	1335	1264	1335	1406
12	1/36	668	617	617	719

Chi-squared statistic = 13.5 Not significant p=0.20

3.3 Matching Pairs

Pair	Probability	Expected	Lower	Sample	Upper
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1+1	1/36	668	617	636	719
2+2	1/36	668	617	681	719
3+3	1/36	668	617	641	719
4+4	1/36	668	617	670	719
5+5	1/36	668	617	669	719
6+6	1/36	668	617	617	719

Chi-squared statistic = 6.7 Not significant p=0.25

3.4 Un-matching Pairs

First	Second	Probability	Expected	Lower	Sample	Upper
1	2	1/36	668	617	669	719
1	3	1/36	668	617	673	719
1	4	1/36	668	617	703	719
1	5	1/36	668	617	665	719
1	6	1/36	668	617	606	719
2	1	1/36	668	617	642	719
2	3	1/36	668	617	675	719
2	4	1/36	668	617	703	719
2	5	1/36	668	617	679	719
2	6	1/36	668	617	706	719
3	1	1/36	668	617	673	719
3	2	1/36	668	617	686	719
3	4	1/36	668	617	676	719
3	5	1/36	668	617	715	719
3	6	1/36	668	617	631	719
4	1	1/36	668	617	660	719
4	2	1/36	668	617	696	719
4	3	1/36	668	617	692	719
4	5	1/36	668	617	652	719
4	6	1/36	668	617	656	719
5	1	1/36	668	617	688	719
5	2	1/36	668	617	682	719
5	3	1/36	668	617	672	719
5	4	1/36	668	617	664	719
5	6	1/36	668	617	673	719
6	1	1/36	668	617	651	719
6	2	1/36	668	617	675	719
6	3	1/36	668	617	657	719
6	4	1/36	668	617	639	719
6	5	1/36	668	617	662	719

Chi-squared statistic = 24.7 Not significant p=0.70



3.5 Two Identical Rolls

The identical rolls have been lumped because the sample size is too small for individual sequences.

Pairs	Probability	Expected	Lower	Sample	Upper
Any of 1+1 ; 1+1 2+2 ; 2+2 3+3 ; 3+3 4+4 ; 4+4 5+5 ; 5+5 6+6 ; 6+6	1/216	99	80	118	119

Notes:

- a) In the above tables, 1 number is shown as bold. This number is outside the lower limit. Up to 5% of the numbers can be outside the limits.

Comments:

In the above tables for Board specific calculations, 98.15% of actual values lie within the lower and upper limits. None of the table has a significant p-value. This shows that the Random Number Generator (RNG) is working correctly.

4 Conclusion

Probabilities for Any Specific Number: These calculations indicated statistical randomness. The chi-squared statistic and the p-value were within acceptable limits. The observed values were within 95% confidence limits of the theoretical values.

Probabilities for Sum of Pairs: These calculations indicated statistical randomness. The chi-squared statistic and the p-value were within acceptable limits. The observed values were within 95% confidence limits of the theoretical values.

Probabilities for Matching Pairs: These calculations indicated statistical randomness. The chi-squared statistic and the p-value were within acceptable limits. The observed values were within 95% confidence limits of the theoretical values.

Probabilities for Un-matching Pairs: These calculations indicated statistical randomness. The chi-squared statistic and the p-value were within acceptable limits. The observed values were within 95% confidence limits of the theoretical values.

Probabilities for Identical Rolls: These calculations indicated statistical randomness. The observed values were within 95% confidence limits of the theoretical values.

In all the above chi-squared tests, none was found to be statistically significant. There is thus no evidence of non-randomness in the sequence of dice throws. iTech Labs concludes that the Random Number Generator used for the games in the PartyGammon system is operating correctly and the dice numbers are generated in a fair manner.

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